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* Diameter of a set

Let A be a subset of the metric space (X, D). Then the diameter of A is the lub (least upper bound) (i.e. Supremum) of the set of all distances between points of A. It is denoted by $\delta(A)$ or $d(A)$

$$\therefore \delta(A) \text{ or } d(A) = \text{lub}\{d(x,y) : x, y \in A\}.$$

* Sequence in a metric space

Let (X,d) be a metric space. Then a sequence S or $\mathcal{S}$ in X is a function from set N (of natural numbers) into X

i.e $\mathcal{S} : N \to X$.

The image of natural number $n \in N$ under the sequence $\mathcal{S}(or S)$ is denoted by S_n or $S(n)$. S_n is, in general, called n^{th} term of the sequence.

It is denoted as $\langle S_n \rangle$ or $\{S_n\}$ or $\langle S_1, S_2, \dots S_n \rangle$

Impt

The terms of the sequence may be identical. i.e. They need not be distinct.

* Range set of a sequence

Range set X of a sequence may be finite or infinite.

Example — (i) for the sequence $S_n = \langle (-1)^n \rangle$, range set $= \{-1, 1\}$ which is finite.

(ii) For the sequence $S_n = \frac{n}{n+1}$,

i.e. the sequence $\langle \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots \rangle$ the range is the infinite set

$\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\}$.

Important 1. The diameter of a set may be finite/infinite according as $d(A)$ is a real number or $\pm\infty$.

2. An empty set has infinite diameter since $d(\phi) = -\infty$.

3. A bounded set has finite diameter.

* A mapping of a non-empty set into a metric space is called a Bounded mapping if its range is a bounded set.